McGill University
Department of Mathematics and Statistics

Ph.D. preliminary examination, PARTami0n6y

Single variable real analysis

Vector calculus, ODE, and complex analysis

Solve any three out of the four questions 9, 10, 11, and 12.

Problem 9. Find a general solution of the equation

$$y'''' \quad 5y'' + 4y = 80e^{3x}:$$

Problem 10. Find at least three nonzero terms in the power series expansion (about $x = 0$) of solutions to the equations $xy'' + (2 \quad x)y' \quad y = 0:$

Problem 11. Is it possible to solve the system

$$
\begin{cases}
xy^2 + xzu + yv^2 = 3; \\
u^3 yz + 2xv \quad u^2 v^2 = 2:
\end{cases}
$$

for $(u; v)$ as functions of $(x; y; z)$ near $(x; y; z) = (1; 1; 1)$ and $(u; v) = (1; 1)$? If so, compute the Jacobian $@(u; v) = @(x; y; z)$.

Problem 12. Compute $\iint_S |xyz| dS;$ where S is a part of $z = x^2 + y^2$ bounded by the plane $z = 1$.